J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

SYJC TEST - 05 - SET 1

DURATION - 1 $^1/_2$ HR **MARKS - 40**

TOPIC : SECTION–1 INTEGRATION (INDEFINITE & DEFINITE) , APP. OF DEFINITE INTEGRALS

SECTION–II SEQUENCING , ASSIGNMENT , DEMOGRAPHY

SOLUTION SET

SECTION - I

Q1. Attempt any THREE of the following (2 marks each) **(6 marks)**

01. $\displaystyle\int \frac{1}{x(3 + \log x)}\, dx$

SOLUTION PUT

$3 + \log x = t$

$\frac{1}{x} . dx = dt$

NOW THE SUM IS

$= \displaystyle\int \frac{1}{t}\, dt$

$= \log | t | + c$

RESUBS.

$= \log | 3 + \log x | + c$

02. $\displaystyle\int \frac{1}{x^2 + 8x + 20}\, dx$

Q1

SOLUTION $\left(\frac{1}{2}(8)\right)^2 = 16$

$= \displaystyle\int \frac{1}{x^2 + 8x + 16 + 20 - 16}\, dx$

$= \displaystyle\int \frac{1}{(x + 4)^2 + 4}\, dx$

$= \displaystyle\int \frac{1}{(x + 4)^2 + 2^2}\, dx$

$= \frac{1}{2} \tan^{-1} \frac{x + 4}{2} + c$

03. $\displaystyle\int \frac{1}{x\left[(\log x)^2 + 4\right]}\, dx$

Q1

SOLUTION

PUT $\log x = t$

$\frac{1}{x} \cdot dx = dt$

THE SUM IS

$$= \int \frac{1}{t^2 + 4}\, dt$$

$$= \int \frac{1}{t^2 + 2^2}.\, dt$$

$$= \frac{1}{a} \tan^{-1} \frac{t}{a} + c$$

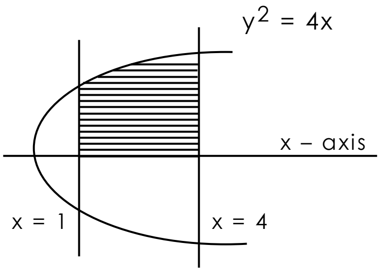
$$= \frac{1}{2} \tan^{-1} \frac{t}{2} + c$$

Resubs.

$$= \frac{1}{2} \tan^{-1} \left(\frac{\log x}{2}\right) + c$$

04. Find the area of the region bounded by the curve $y^2 = 4x$ and the lines $x = 1$; $x = 4$ and the x – axis

SOLUTION



$$A = \int_1^4 y \, dx$$

$$= \int_1^4 \sqrt{4x}\, dx$$

$$= \int_1^4 2\sqrt{x}\, dx$$

Q1

$$= 2 \int_{1}^{4} x^{1/2}\, dx$$

$$= 2 \left[\frac{x^{3/2}}{\frac{3}{2}} \right]_{1}^{4}$$

$$= \frac{4}{3} \left[x^{3/2} \right]_{1}^{4}$$

$$= \frac{4}{3} \left[4^{3/2} - 1^{3/2} \right]$$

$$= \frac{4}{3} \left[2^{2.3/2} - 1 \right]$$

$$= \frac{4}{3} \left[2^{3} - 1 \right]$$

$$= \frac{4}{3} (8 - 1)$$

$$= \frac{28}{3} \text{ sq. units}$$

Q2. Attempt any TWO of the following (3 marks each) **(6 marks)**

Q2

01. $\int e^x \dfrac{x + 3}{(x + 4)^2}.\, dx$

SOLUTION

$$\int e^x \left(\frac{x + 3}{(x + 4)^2}\right) dx$$

$$= \int e^x \left(\frac{x + 4 - 1}{(x + 4)^2}\right) dx$$

$$= \int e^x \left(\frac{x + 4}{(x + 4)^2} - \frac{1}{(x + 4)^2}\right) dx$$

$$= \int e^x \left(\frac{1}{x + 4} + \frac{-1}{(x + 4)^2}\right) dx$$

$$\frac{d}{dx}\frac{1}{x + 4} = \frac{-1}{(x + 4)^2}$$

$$= e^x \quad f(x) + f'(x) \quad dx$$

$$= \int e^x \left(f(x) \quad + \quad c\right)$$

$$= \frac{e^x}{x + 4} + c$$

02. $\int \tan^{-1}x \, dx$

SOLUTION

$$= \tan^{-1}x \int 1\, dx - \int\left(\frac{d}{dx} \tan^{-1}x \int 1\, dx\right)dx$$

$$= \tan^{-1}x \,.\, x - \int \frac{1}{1 + x^2}.\, x \quad dx$$

$$= x.\tan^{-1}x - \int \frac{x}{1 + x^2}.\, dx$$

$$= x.\tan^{-1}x - \frac{1}{2}\int \frac{2x}{1 + x^2}.\, dx$$

$$= x.\tan^{-1}x - \frac{1}{2} \log|1 + x^2| + c$$

03. $\int \frac{\cos x}{\sqrt{9 - 8\sin x - \sin^2 x}} \, dx$

Q2

SOLUTION PUT $\sin x = t$

$\cos x \, . \, dx = dt$

THE SUM IS

$$\int \frac{1}{\sqrt{9 - 8t - t^2}} \, dt$$

$$= \int \frac{1}{\sqrt{9 - (t^2 + 8t)}} \, dt$$

$$= \int \frac{1}{\sqrt{9 - (t^2 + 8t + 16) + 16}} \, dt$$

$$= \int \frac{1}{\sqrt{25 - (t + 4)^2}} \, dt$$

$$= \int \frac{1}{\sqrt{5^2 - (t + 4)^2}} \, dt$$

$$= \sin^{-1} \frac{t}{a} + c$$

$$= \sin^{-1} \left(\frac{t + 4}{5} \right) + c$$

$$= \sin^{-1} \left(\frac{\sin x + 4}{5} \right) + c$$

Q3. Attempt any TWO of the following (4 marks each) **(8 marks)**

Q3

01. $\displaystyle\int \frac{1 + \log x}{x(2 + \log x)(3 + \log x)}\, dx$

SOLUTION

$\log x = t \qquad \therefore \frac{1}{x} . dx = dt$

$$= \int \frac{1 + t}{(2 + t)(3 + t)}\, dt$$

$$\frac{1 + t}{(2 + t)(3 + t)} = \frac{A}{2 + t} + \frac{B}{3 + t}$$

$$1 + t = A(3 + t) + B(2 + t)$$

Put $t = -3$

$$1 - 3 = B(2 - 3)$$

$$-2 = B(-1) \qquad \therefore B = 2$$

Put $t = -2$

$$1 - 2 = A(3 - 2)$$

$$-1 = A(1) \qquad \therefore A = -1$$

HENCE

$$\frac{1 + t}{(2 + t)(3 + t)} = \frac{-1}{2 + t} + \frac{2}{3 + t}$$

BACK IN THE SUM

$$= \int \frac{-1}{2 + t} + \frac{2}{3 + t}\, dt$$

$$= -\log|2 + t| + 2\log|3 + t| + c$$

RESUBS. $\quad = -\log|2 + \log x| + 2\log|3 + \log x| + c$

02. $\displaystyle\int_{3}^{9} \frac{\sqrt[3]{12-x}}{\sqrt[3]{x} + \sqrt[3]{12-x}}\, dx$

Q3

SOLUTION

$$I = \int_{3}^{9} \frac{\sqrt[3]{12-x}}{\sqrt[3]{x} + \sqrt[3]{12-x}}\, dx \quad \textbf{.......(1)}$$

USING $\displaystyle\int_{a}^{b} f(x)dx = \int_{b}^{b} f(a+b-x)\, dx$

$$I = \int_{3}^{9} \frac{\sqrt[3]{12-(12-x)}}{\sqrt[3]{12-x} + \sqrt[3]{12-(12-x)}}\, dx$$

$$I = \int_{3}^{9} \frac{\sqrt[3]{12-\ 12+x}}{\sqrt[3]{12-x} + \sqrt[3]{12-12+x}}\, dx$$

$$I = \int_{3}^{9} \frac{\sqrt[3]{x}}{\sqrt[3]{12-x} + \sqrt[3]{x}}\, dx \quad \textbf{.......(2)}$$

(1) + (2)

$$I = \int_{3}^{9} \frac{\sqrt[3]{12-x} + \sqrt[3]{x}}{\sqrt[3]{12-x} + \sqrt[3]{x}}\, dx$$

$$2I = \int_{3}^{9} 1\ dx$$

$$2I = \Big(x \Big)_{3}^{9}$$

$$2I = 9 - 3$$

$$2I = 6$$

$$I = 3$$

03. Find the volume of a solid obtained by the complete revolution of the ellipse

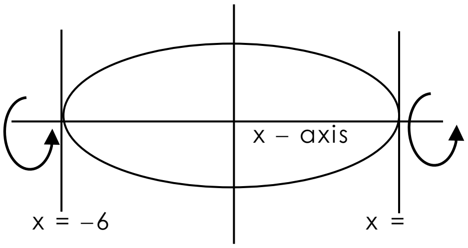
$$\frac{x^2}{36} + \frac{y^2}{25} = 1 \qquad \textbf{about x – axis}$$

–6

SOLUTION

STEP 1 :

$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

6

$a^2 = 36$; $a = 6$

$b^2 = 25$, $b = 5$

STEP 2 :

$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

$$\frac{y^2}{25} = 1 - \frac{x^2}{36}$$

$$\frac{y^2}{25} = \frac{36 - x^2}{36}$$

$$y^2 = \frac{25}{36}(36 - x^2)$$

STEP 3 :

$$V = \pi \int_{-6}^{6} y^2.dx$$

$$= \pi \int_{-6}^{6} \frac{25}{36}(36 - x^2).dx$$

$$= \frac{25\pi}{36} \int^{6} (36 - x^2).dx$$

$$= \frac{25\pi}{36}\left[36x - \frac{x^3}{3}\right]_{-6}^{6}$$

$$= \frac{25\pi}{36}\left\{\left(216 - \frac{216}{3}\right) - \left(-216 + \frac{216}{3}\right)\right\}$$

$$= \frac{25\pi}{36}\left\{\left(216 - 72\right) - \left(-216 + 72\right)\right\}$$

$$= \frac{25\pi}{36}\left\{\left(144\right) - \left(-144\right)\right\}$$

$$= \frac{25\pi}{36}\left(288\right)$$

$= 200\,\pi$ cubic units

SECTION - II

Q4. Attempt any THREE of the following (2 marks each) **(6 marks)**

Q4

01. Compute Age – Specific Death rate for the following data

AGE GROUP	NO. OF PERSONS	NO. OF DEATHS	**SDR = $\frac{D}{P}$ X 1000**
0 – 20	7000	140	$\frac{140}{7000} \times 1000 = 20$
20 – 25	20000	180	$\frac{180}{20000} \times 1000 = 9$
25 – 65	10000	120	$\frac{120}{10000} \times 1000 = 12$
65 & above	4000	160	$\frac{160}{4000} \times 1000 = 40$

02. For the following problem , **find the sequence** that minimizes total elapsed time required to complete the following jobs on two machines M_1 & M_2 in the order $M_1 - M_2$

Jobs	A	B	C	D	E
Machine M_1	5	1	9	3	10
Machine M_2	2	6	7	8	4

Min time = 1 on job B on machine M_1 . Place the job at the start of the sequence

B				

Next min time = 2 on job A on machine M_2 . Place the job at the end of the sequence

B				A

Next min time = 3 on job D on machine M_1 . Place it at the start of the sequence after B

B	D			A

Next min time = 4 on job E on machine M_2 . Place it at the end of the sequence before A

B	D		E	A

OPTIMAL SEQUENCE

B	D	C	E	A

03. in a complete life table $l_4 = 60$ and $L_4 = 45$. Find the value of p_4

Q4

STEP 1 :

$$L_x = \frac{l_x + l_{x+1}}{2}$$

$$L_4 = \frac{l_4 + l_5}{2}$$

$$45 = \frac{60 + l_5}{2}$$

$$90 = 60 + l_5$$

$$\mathbf{l_5 = 30}$$

STEP 2 :

$$p_x = \frac{l_{x+1}}{l_x}$$

$$p_4 = \frac{l_5}{l_4}$$

$$= \frac{30}{60}$$

$$\mathbf{p_4 = 0.5}$$

04. **SOLUTION**

Age Group	Population	No. of deaths
0 – 20	40000	350
20 – 65	65000	650
65 & above	15000	x

$\Sigma P = 120000$ $\Sigma D = 1000 + x$

$$CDR = \frac{\Sigma D \times 1000}{\Sigma P}$$

$$13.4 = \frac{1000 + x}{120\cancel{000}} \times \cancel{1000}$$

$$1608 = 1000 + x$$

$$x = 608$$

Q5. **Attempt any TWO of the following** (3 marks each) **(6 marks)**

Q5

01. Complete the following life table

x	l_x	d_x	q_x	p_x	L_x
4	9100	60	?	?	?
5	?	45	?	?	

SOLUTION

$$d_x = l_x - l_{x+1}$$

$d_4 = l_4 - l_5$	$d_4 = l_4 - l_5$
$60 = 9100 - l_5$	$45 = 9040 - l_6$
$l_5 = 9100 - 60$	$l_6 = 9040 - 45$
$l_5 = 9040$	$l_6 = 8995$

$$q_x = \frac{d_x}{l_x}$$

$$q_4 = \frac{d_4}{l_4} = \frac{60}{9100} = 0.0066$$

LOG CALC
1.7782
− 3.9590
AL $\bar{3}.8192$
0.006595

$$q_5 = \frac{d_5}{l_5} = \frac{45}{9040} = 0.0050$$

LOG CALC
1.6532
− 3.9562
AL $\bar{3}.6970$
0.004977

$$p_x = 1 - q_x$$

✓ $p_4 = 1 - q_4 = 1 - 0.0066 = 0.9934$

✓ $p_5 = 1 - q_5 = 1 - 0.0050 = 0.9950$

$$L_x = \frac{l_x + l_{x+1}}{2}$$

✓ $L_4 = \frac{l_4 + l_5}{2} = \frac{9100 + 9040}{2} = 9070$

✓ $L_5 = \frac{l_5 + l_6}{2} = \frac{9040 + 8995}{2} = \frac{18035}{2} = 9017.5$

Q5

02. Calculate CDR for district A and B and compare

SOLUTION

Age Group (Years)	DISTRICT A		DISTRICT B	
	NO. OF PERSONS IN '000 **P**	NO. OF DEATHS **D**	NO. OF PERSONS IN '000 **P**	NO. OF DEATHS **D**
0 – 15	1	20	2	50
15 – 55	3	30	7	70
Above 55	2	40	1	25
	Σ **P = 6**	Σ **D = 90**	Σ **P = 10**	Σ **D = 145**

$$CDR(A) = \frac{\Sigma D}{\Sigma P} = \frac{90}{6} = 15$$

(DEATHS PER THOUSAND)

$$CDR(B) = \frac{\Sigma D}{\Sigma P} = \frac{145}{10} = 14.5$$

(DEATHS PER THOUSAND)

COMMENT : CDR(B) < CDR(A) . HENCE DISTRICT B IS HEALTHIER THAN DISTRICT A

03. the departmental store has four workers to pack their items . The timings in minutes required for each worker to complete the packings per item sold is given below . How should the manager of the store assign the jobs to the workers , so as to minimize the total time of packing

		Items			
		Books	Toys	Crockery	cutlery
Workers	A	2	10	9	7
	B	13	2	12	2
	C	3	4	6	1
	D	4	15	4	9

0	8	7	5
11	0	10	0
2	3	5	0
0	11	0	5

Reducing the matrix using row minimums

Allocation Uisng SINGLE ZERO ROW COLUMN METHOD

$\boxed{0}$	8	7	5
11	$\boxed{0}$	10	$\cancel{0}$
2	3	5	$\boxed{0}$
$\cancel{0}$	11	$\boxed{0}$	5

Optimal Assignment

A – Books ; B – Toys ; C – Cutlery ; D – Crockery

Minimum time = 2 + 2 + 4 + 1 = 9 minutes

Q6. Attempt any TWO of the following (4 marks each) **(8 marks)**

Q6

01.

AGE x	lx	$dx = lx - lx{+}1$	$qx = \frac{dx}{lx}$	$px = 1 - qx$	$Lx = \frac{lx + lx{+}1}{2}$	Tx	$e_x^0 = \frac{Tx}{lx}$
0	1000	1000 − 850 = 150	$\frac{150}{1000} = 0.15$	1 − 0.15 = 0.85	850 + 75 = 925	2495	$\frac{2495}{1000} = 2.495$
1	850	850 − 760 = 90	$\frac{90}{850} = 0.1059$	1 − 0.1059 = 0.8941	760 + 45 = 805	1570	$\frac{1570}{850} = 1.847$
2	760	760 − 360 = 400	$\frac{400}{760} = 0.5264$	1 − 0.5264 = 0.4736	360 + 200 = 560	765	$\frac{765}{760} = 1.007$
3	360	360 − 25 = 335	$\frac{335}{360} = 0.9305$	1 − 0.9305 = 0.0695	25+ 167.5= 192.5	205	$\frac{205}{360} = 0.5696$
4	25	25 − 0 = 25	$\frac{25}{25} = 1$	1 − 1 = 0	0 + 12. 5 = 12.5	12.5	$\frac{12.5}{25} = 0.5$
5	0	----	----	----	----	----	----

LOG CALCULATIONS FOR 'qx'

LOG 90 − LOG 850	LOG 400 − LOG 760	LOG 335 − LOG 360
1.9542	2.6021	2.5250
− 2.9294	− 2.8808	− 2.5563
AL $\bar{1}.0248$	AL $\bar{1}.7213$	AL $\bar{1}.9687$
0.1059	0.5264	0.9305

LOG CALCULATIONS FOR 'e_x^0'

LOG 1570 − LOG 850	LOG 765 − LOG 760	LOG 205 − LOG 360
3.1959	2.8837	2.3118
− 2.9294	− 2.8808	− 2.5563
AL 0.2665	AL 0.0029	AL $\bar{1}.7555$
1.847	1.007	0.5696

02. Find the sequence that minimizes total elapsed time (in hours) required to complete the following jobs on two machines M_1 and M_2 in the order $M_1 M_2$. Also find the minimum elapsed time and idle time for two machines

Q6

Job	A	B	C	D	E	F
M_1	5	9	4	7	8	6
M_2	7	4	8	3	9	5

Step 1 : Finding the optimal sequence

Min time = 3 on job D on machine M_2 . Place the job at the end of the sequence

					D

Next min time = 4 on job B on machine M_2 & on job C on machine M_1 . Place the job B at the end of the sequence before D & job C at the start of the sequence sequence

C				B	D

Next min time = 4 on job A on machine M_1 & on job F on machine M_2 . Place the job A at the start of the sequence after C & job F at the end of the sequence before B

C	A		F	B	D

OPTIMAL SEQUENCE

C	A	E	F	B	D

Step 2 : Work table

According to the optimal sequence

Job	C	A	E	F	B	D	**total process time**
M_1	4	5	8	6	9	7	= 39 hrs
M_2	8	7	9	5	4	3	= 36 hrs

WORK TABLE

	MACHINES				
	M_1		M_2		Idle time
JOBS	**IN**	**OUT**	**IN**	**OUT**	**on M_2**
C	0	4	4	12	4
A	4	9	12	19	
E	9	17	19	28	
F	17	23	28	33	
B	23	32	33	37	
D	32	39	39	42	2

Step 3 :

Total elapsed time **T** = 42 hrs

Idle time on M_1 = $T - \left(\text{sum of processing time of all jobs on M1} \right)$

$= 42 - 39$

$= 3$ hrs

Idle time on M_2 = $T - \left(\text{sum of processing time of all jobs on M2} \right)$

$= 42 - 36$

$= 6$ hrs (**CHECK** : 4 + 2 = 6)

Q6

03. a pharmaceutical company has four branches , one each at city A , B , C & D . A branch manager is to be appointed one at each city , out of four candidates P ,Q , R and S . The monthly business depending upon the city and the effectiveness of the branch manager in that city is given below

Branch Manager	Monthly Business (in lacs) A	B	C	D
P	10	10	8	8
Q	12	15	10	9
R	11	16	12	7
S	15	13	15	11

6	6	8	8
4	1	6	7
5	0	4	9
1	3	1	5

substracting all the elements in the matrix from the largest value '16'

the matrx can now be solved for 'MINIMAL ASSIGNMNET PROB'

0	0	2	2
3	0	5	6
5	0	4	9
0	2	0	4

reducing the matrix using 'ROW MINIMUM'

0	0	2	0
3	0	5	4
5	0	4	7
0	2	0	2

reducing the matrix using 'COLUMN MINIMUM'

$\cancel{0}$	$\cancel{0}$	2	$\boxed{0}$
3	$\boxed{0}$	5	4
5	$\cancel{0}$	4	7
$\cancel{0}$	2	$\boxed{0}$	2

Allocation using 'SINGLE ZERO ROW COLUMN METHOD'

Allocation Incomplete

Drawing minimum no. of lines to cover all zero's

0	3	2	0
0	0	2	1
2	0	1	4
0	5	0	2

Revise the matrix -

Reduce all the UNCOVERED elements by its minimum '3' and ADD the same at the INTERSECTION

$\cancel{0}$	3	2	$\boxed{0}$
$\boxed{0}$	$\cancel{0}$	2	1
2	$\boxed{0}$	1	4
$\cancel{0}$	5	$\boxed{0}$	2

Reallocation using 'SINGLE ZERO ROW COLUMN METHOD'

Since all rows now contain an assigned zero , the assignment problem is COMPLETE

OPTIMAL ASSIGNMENT

P – D , Q – A , R – B , S – C , maximum business = 51 lacs